Spherically symmetric space-time defect solution of Einstein field equations.

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Abstract

A new class of spacetime defect solutions of Einstein Field equations of Edelen's direct Poincaré Gauge Field theory without biaxial symmetry is presented. The interior solution describes a core of defects where curvature vanishes and Cartan torsion is nonvanishing. Outside the core (in vacuum) the solution represents a spacetime with vanishing curvature and torsion describing a nontrivial topological defect solution of Einstein equations of gravity. Our solution corresponds to a very weak strenght of Tachyons can be found far away from the core defect.

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1 Introduction

Recently D.G.B.Edelen [1] has presented a class of spacetime defect solutions of Einstein field equations of general relativity with biaxial symmetry where defect core functions are homogeneous of degree - 2 yielding a vanishing torsion. Riemann curvature is assumed to vanish and therefore the solution is a solution of Einstein field equation in vacuum $R_{ik} = 0$, although being topological nontrivial. In this paper I shall be concerned with the investigation of a new class of defect solutions of Einstein field equations without biaxial symmetry. Inside the defect core curvature vanishes and only Cartan's torsion is nonzero. More recently several [2, 3, 4, 5, 6] spacetime defect metrics have been given as solutions of Einstein-Cartan field equations of general relativity with spin and torsion. One of these solutions describes defects in Weitzenböck spacetime [5]. We also compute the geodesics for the vacuum part of the solution and show that there is a tachyonic sector as in previously Edelen [1] paper.

2 Gauge Differential Geometry.

In 1986 and 1989 Edelen described the following results. The Minkowski spacetime M_4 with global coordinates $\{x, y, z, t\}$ is the base of a L_4 Riemann-Cartan Spacetime which is generated from the action of a Poincaré group on M_4. The Riemann-Cartan manifold is , in general, endowed with both curvature and torsion. The translation group T(4) yields the compensating one-forms $\phi^i = \phi^i{}_j(x^k)dx^j \quad (1 \leq i \leq 4)$ and local axial of the six-parameter L(6) local Lorentz group and ten-parameters Poincaré group P(10)$\subset$GL(5,R)are given by

$$W^\alpha = W^\alpha{}_i(x^k)dx^i \qquad (1 \leq \alpha \leq 6) \tag{1}$$

$$B^i = B^i{}_j(x^k)dx^j = (\delta^i{}_j + W^\alpha{}_j l^i{}_{k\alpha} x^k + \phi^i{}_j)dx^j \tag{2}$$

respectively. The distortion 1-forms $\{B^i | 1 \leq i \leq 4\}$ are the basis of a vector space $\wedge^1$ of forms on L_4.

The distorted Riemann-Cartan spacetime L_4 obtained from M_4 by minimal substitution yields the line element

$$ds^2 = g_{ij} dx^i \otimes dx^j \tag{3}$$

where $g_{ij} = B^r{}_i h_{rs} B^s{}_j, g = det(g_{ij}) = -B^2$ and $ds^2 = h_{ij} dx^i \otimes dx^j$ is the M_4 line element.

The spacetime L_4 has both curvature and torsion in general. The Cartan torsion 2-forms $\{\sum^i | 1 \leq i \leq 4\}$ are given by

$$\sum{}^i = dB^i + W^\alpha l^i{}_{j\alpha} \wedge B^j \tag{4}$$

Where the holonomic torsion 2-forms $S^k = \frac{1}{2}(\Gamma^k{}_{ij} - \Gamma^k{}_{ji}) dx^i \wedge dx^j$ are determined in terms of the $\sum^i$ by $S_k = b^k{}_r \sum^r$ where $b_i \rfloor B^j = \delta^j{}_i$, $b_i = b^j{}_i(x^k)\partial_j$ being the frames of B^j . In general the torsion forms are given by (the coframes)

$$\sum{}^i = \theta^\alpha l^i{}_{j\alpha} \chi^j + d\phi^i + W^\alpha l^i{}_{j\alpha} \wedge \phi^j \tag{5}$$

where $\theta^\alpha = \frac{1}{2}\theta^\alpha{}_{rs} dx^r \wedge dx^s$ and the Riemann curvature is given by $R^i{}_{rsj} = \theta^\alpha{}_{rs} L^i{}_{j\alpha}$. In this paper we shall be concerned with dislocations where curvature vanishes and only torsion survives. Thus $\theta^\alpha = 0$, $R^i{}_{rsj} = 0$. Defining $W^\alpha \equiv 0$ the dislocation density and current (Cartan torsion) reduces to $\sum^i = d\phi^i$ and the distortion 1-forms have the form $B^i = dx^i + \phi^i$. In general in crystalline solids the procedure consists in giving the dislocation density 2-forms and then to calculate the response of the solid. Here we shall consider a dislocation density like

$$\sum{}^i = A^i(R, t) dR \wedge dt \tag{6}$$

From the expression $\sum^i = d\phi^i$, $d\sum^i = 0$. On integration of the system yields

$$\phi^i = a^i(R, t)(Rdt - tdR) \tag{7}$$

the essential difference between these functions here and Edelen's functions in [1] is that the functions here are not biaxial functions [7]. The functions (7) are indeed homogeneous of degree -2 outside the core of defects since Cartan torsion is

$$\sum{}^i = d\phi^i = \{\frac{\partial a^i}{\partial R} R + \frac{\partial a^i}{\partial t} t + 2a^i\} dR \wedge dt \tag{8}$$

and therefore the region $R > R_0$ (here $R = \sqrt{x^2 + y^2 + z^2}$ is a homogeneous function of degree 1) if a^i are homogeneous of degree -2, $\sum^i = 0$ from (8) and curvature and torsion vanish. Despite of this situation the solution of Einstein field equation in vacuum ($R_{ik} = 0$) is topologically nontrivial like the ones [9] obtained earlier by Marder in the context of general relativity and by Tod [8] and Letelier [3, 4] in the context of Einstein-Cartan theory of gravity.

3 Spherically Symmetric Dislocation in Space-time.

To obtain the metric form of the above solution and to investigate the geodesics one needs to compute the frame $\{b^i\}$ basis which yields (here we have consider the approximation where $O(f^2) \to 0$, where the strenght of dislocation is very weak)

$$\begin{aligned} B^1 &= -(1 - fR)dR - f\frac{R}{t}dt & (9) \\ B^2 &= d\theta + a \quad , \quad B^3 = d\varphi & (10) \\ B^4 &= (1 + fR)dt - ftdR & (11) \end{aligned}$$

and

$$\begin{aligned} b_1 &= -(1 + fR)\partial_R + ft\partial_t & (12) \\ b_2 &= \partial_\theta \quad , \quad b_3 = \partial_\varphi & (13) \\ b_4 &= -\frac{fR}{t}\partial_R + (1 - fR)\partial_t & (14) \end{aligned}$$

where we have used the result $a^1 \equiv -f$ and $Ra^1 = ta^4$. From the frame equations we obtains the line element

$$ds^2 = +(1+fR)^2dt^2-(1-fR)^2dR^2-R^2(d\theta^2+\sin^2\theta d\varphi^2)-2ft(1+\frac{1}{2}Rf+ft)dRdt \quad (15)$$

Notice that for $f \equiv -\frac{GM}{R^2}$ and $O(f^2) \to 0$,(15) reduces to Schwarzschild metric. Nevertheless this is not a solution to our problem since Shwarzschild solution although is a solution of Einstein field equation in vacuum has a nonvanishing Riemann curvature. Sphericalbubbles of this type have been consider by Letelier and Wang [10]. In our case one must define $f \equiv \frac{K}{R^2}$ where K is a dislocation strength being zero outside the core defect. Therefore f must vanish outside the core defect and the metric (15) will be flat outside the core defect. This metric is nonsingular since $det(g_{ij}) \neq 0$ as can be easily checked. Metric (15) fits into the general spherically symmetric form

$$ds^2 = A(r,t)dt^2 - B(r,t)dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) - 2F(r,t)drdt \tag{16}$$

as it is known from Riemannian geometry this metric can be reduced to a static metric by a change of coordinates. Therefore we are left with a spherically symmetric solution of the Einstein field equation. To compute the geodesic equations

$$\dot{v}^i = 0 \qquad\qquad \dot{u}^i = b^i{}_j v^j \tag{17}$$

Let us define the vector v^i from the first equation in (17) as

$$v^i = (0, k, 0, 1) \tag{18}$$

where k is a constant. From the velocity expression $V^i = b^i{}_j v^j$ and (18) one obtains

$$V^i = (0, k, 0, (1 - fR)) \tag{19}$$

To obtain the geodesic equations we substitute (19) into the equations

$$v^i = B^i{}_k V^k \tag{20}$$

and use these into the second eqn. in (17) obtaing

$$\frac{dR}{d\tau} = ft \tag{21}$$

$$\frac{dt}{d\tau} = (1 - fR) \tag{22}$$

$$\frac{d\theta}{d\tau} = k \quad , \quad \frac{d\varphi}{d\tau} = 0 \tag{23}$$

Since our aim is to show the existence of tachyons even in the linear approximation of the strenght of dislocation $K(O(K^2) > 0)$ we made the simplest choice for the torsion function f or $f \equiv \frac{K}{R^2}$. From metric (15) one may notice that the solution $f \equiv K$ would led to a non-Minkovski metric as R goes to infinite. With this choice the geodesic equations can be rewritten as

$$\frac{d^2R}{d\tau^2} = K(\frac{1}{R} - \frac{K\tau}{R^2}) \quad (R(0) = R_0) \tag{24}$$

$$\frac{d^2t}{d\tau^2} = \frac{K^2 t}{R^4} \approx 0 \tag{25}$$

$$\frac{d^2\theta}{d\tau^2} = 0 \quad , \quad \frac{d^2\varphi}{d\tau^2} = 0 \tag{26}$$

Which integrate to

$$R(\tau) = \frac{K\tau^2}{2} + R_0 \tag{27}$$

$$t(\tau) = \tau \quad , \quad \theta(\tau) = K \quad , \quad \varphi(\tau) = const. \tag{28}$$

$$V^2 \equiv V^i g_{ij} V^j = v^i h_{ij} v^j = (1 - k^2 R^2) > 0 \tag{29}$$

thus this observer is a proper test particle for the spacetime L_4.

Notice that an observer in the asymptotic Minkovski space at infinity would obtain $V^i h_{ij} V^j \cong [1 - 2KR_0(1 - \frac{k^2 R_0}{2K})]$ around $\tau = 0$. The spatial part of thus velocity would be

$$V^2 = 2KR_0(1 - \frac{k^2 R_0}{2K}) \tag{30}$$

From formula (30) it is easy to note that the region $R_0 = \frac{2K}{k^2}$ is forbidden for tachyons. Around this spherical surface tachyons are not forbidden. Thus there is a possibility to find tachyons around weak spherical spacetime defect cores. Letelier and Wang [10, 11] have investigated spherically symmetric spacetime defects without torsion where Riemann-Christoffel is novanishing only at surface defects. In Letelier-Wang's [10] spherical defects no tachyons appear. Another defect solutio given by $f = \frac{K}{R^2}$ would led us to tachyons around the core defect. Other defect geometries and their relation to tachyons can appear elsewhere.

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